2nd National Conference on Soft Computing of Engineering Science in Industry and Society (ASEIS 2023)

۲ همایش ملی محاسبات نرم علوم مهندسی در صنعت و جامعه

A new multi-criteria decision making method on N-sof sets

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Abstract. In this article, we introduce a novel model for multi-criteria decision-making (MCDM). We illustrate our new method with a real life example where multi-criteria index information is described by a N-soft set. The result of this example demonstrate the efficiency and advantage of this method.

Keywords: Multi-criteria decision-making, ELECTRE method, N-soft set.

AMS Mathematics Subject Classification [2010]: 90B50, 91B06

1. Introduction

Many problems in real life involve imprecise data. The solution of these problems requires the mathematical methods that based on imprecision and uncertainty. In recent years, researchers have been proposed a number of theories for dealing with this problems in an effective way, such as fuzzy set theory [5], theory of probability, vague sets, rough set theory, theory of interval mathematics etc. A novel concept of soft theory defined by Molodtsov as a new mathematical tool for dealing with uncertainties [3] . Soft set theory has significant use in many research areas. Fatimah et al. [2] proposed the idea of an extended soft set model, which named N-soft set in order to describe the importance of ordered grades in actually existing problems.

Definition 1.1. Let O be a universe of objects (alternative) and P be a set of parameters (attribute). Let $P(O)$ denote the power set of O and $T \subseteq P$. A pair (F, T) is called a soft set over O, where F is a mapping given by $F : T \to P(O)$. In other words, a soft set over O is a parametrized family of subsets of the universe O. For $\alpha \in T, F(\alpha)$ may be considered as the set of approximate elements of the soft set (F, T).

Definition 1.2. Let O be a universe of objects and P the set of attributes, $T \subseteq P$. Let $G = \{0, 1, 2, 3, ..., N-1\}$ be the set of ordered grades where $N = \{1, 2, ...\}$. A triple (F, T, N) is called an N-soft set on O if F is the mapping $F : T \to 2^{O\times G}$, with property that for each $t \in T$ and $o \in O$ there exists a unique pair $(o, g_t) \in O \times G$ such that $(o, g_t) \in F(t), g_t \in G$.

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Multi-criteria decision-making (MCDM) is one of the most widely used decision methods in the social and medical sciences, engineering, economics etc. MCDM methods can be used to improve the quality of decisions by making the decision-making process more rational and efficient. The typical MCDM problem is concerned with the task of ranking a finite number of decision alternatives, each of which is explicitly described in terms of different characteristics (attributes, decision criteria, or objectives) which have to be taken into account simultaneously. An N-soft set can be presented as a decision matrix as shown in Fig. 1, where a_{ij} and w_j are the grade and weight of alternative A_i in criterion C_j, respectively.

	Criteria			
	C_1	C_2	...	C_n
Alternatives	(w_1	w_2	...	w_n)
A_1	a_{11}	a_{12}	...	a_{1n}
A_2	a_{21}	a_{22}	...	a_{2n}
	.	.	.	
	.	.		.
	.	.		.
A_m	a_{m1}	a_{m2}	...	a_{mn}

Figure 1. Decision matrix.

The ELECTRE methods are based on the evaluation of the concordance and the discordance indices. The concordance index for a pair of alternatives A_i and A_j measures the strength of the hypothesis that alternative A_i is at least as good as alternative A_j. The discordance index measures the strength of evidence against this hypothesis [1]. There are different measures of concordance and discordance indices. In ELECTRE II, the concordance index $C(A_i, A_j)$ for each pair of alternatives (A_i, A_j) is defined as follows:

$$C(A_i, A_j) = \frac{\sum_{i \in Q(A_i, A_j)} w_i}{\sum_{i=1}^{m} w_i},$$

where $Q(A_i, A_j)$ is the set of criteria for which A_i is equal or preferred to (i.e., at least as good as) A_j, and w_i is the weight of the ith criterion.
The discordance index $D(A_i, A_j)$ for each pair of alternatives $D(A_i, A_j)$ is defined as follows:

$$D(A_i, A_j) = \frac{\max_k [a_{jk} - a_{ik}]}{\delta},$$

Where $\delta = max_k |a_{jk} - a_{ik}|$ (i.e., the maximum difference on any criterion).
The proposed discordance index have not enough efficiency and advantage, because it is calculated based on one criterion and another criteria are disregarded. We present a new discordance index which is calculated based on all criteria.

2. New decision making method

In this section, we present a new method to solve MCDM problems. We use the ELECTRE II method to find the best alternative where multi-criteria index information is described by a N-soft set. This method is illustrated using an example. This example is a real-life case study for finding the best location for a wastewater treatment plant in

Ireland [4]. The decision problem is defined on seven criteria and five alternatives. All the criteria are benefit criteria. The decision matrix, that is, the performances of the alternatives A_i in terms of the criteria C_j , is as follows:

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
A_1	1	2	1	5	2	2	4
A_2	3	5	3	5	3	3	3
A_3	3	5	3	5	3	2	2
A_4	1	2	2	5	1	1	1
A_5	1	1	3	5	4	1	5

The weights of the criteria are:

	C_1	C_2	C_3	C_4	C_5	C_6	C_7
$Weight$	0.0780	0.1180	0.1570	0.3140	0.2350	0.0390	0.0590

The concordance indices for this example are as follows:

	A_1	A_2	A_3	A_4	A_5
A_1	1.0000	0.3730	0.4120	0.8430	0.5490
A_2	0.9410	1.0000	1.0000	1.0000	0.7060
A_3	0.9410	0.9020	1.0000	1.0000	0.7060
A_4	0.6670	0.3140	0.3140	1.0000	0.5490
A_5	0.8430	0.7650	0.7650	0.8820	1.0000

The discordance indices $D(A_i, A_j)$ for each pair of alternatives $D(A_i, A_j)$ is defined as follows:

$$D(A_i, A_j) = \frac{\sum_{k=1}^{n} w_k E_{ij}^k (A_i, A_j)}{\eta}, i, j = 1, ..., n$$

where

$$E_{ij}^k (A_i, A_j) = \begin{cases} a_{jk} - a_{ik} & , a_{jk} \geq a_{ik}, \\ 0 & , a_{jk} < a_{ik}, \end{cases}$$

and $\eta = \max_{i,j} \sum_{k=1}^{m} w_k E_{ij}^k (A_i, A_j)$. When the above formula is used, it turns out that the discordance indices for this example are as follows:

	A_1	A_2	A_3	A_4	A_5
A_1	0.0000	0.8237	0.7944	0.1178	0.6324
A_2	0.0443	0.0000	0.0000	0.0000	0.2648
A_3	0.0885	0.0735	0.0000	0.0000	0.3091
A_4	0.3383	1.0000	0.9265	0.0000	0.8237
A_5	0.1178	0.5296	0.5004	0.0885	0.0000

After computing the concordance and discordance indices for each pair of alternatives, two types of outranking relations are used by comparing these indices with the pair of threshold values: (C^*, D^*). The pair (C^*, D^*) is defined as the concordance and discordance thresholds for the strong outranking relation. Then the outranking relations are built according to the following rule:

If $C(A_i, A_j) \geqslant C^*$, $D(A_i, A_j) \leqslant D^*$ and $C(A_i, A_j) \geqslant C(A_j, A_i)$, then alternative A_i is regarded as strongly outranking alternative A_j.

The value of (C^*, D^*) is decided by the decision makers for a particular outranking relation. For this example, the pair of thresholds for the outranking relation is chosen as follows: $C^* = 0.8,\ D^* = 0.2$. According to the above rule, the outranking relations for this example were derived to be as follows:

	A_1	A_2	A_3	A_4	A_5
A_1				$\circledast$	
A_2	$\circledast$		$\circledast$	$\circledast$	
A_3	$\circledast$			$\circledast$	
A_4					
A_5	$\circledast$			$\circledast$	

In the above notation $\circledast$ stands for the outranking relation. For example, $A_1 \circledast A_4$ means that alternative A_1 outranks alternative A_4. On the basis of the outranking relations, then the ascending and descending processes are applied to obtain two complete pre-orders of the alternatives. The details of the processes can be studied in [4].
We build the descending pre-order by starting with the set of “best” alternatives (those which outrank other alternatives) and going downward to the worse one. Also, the ascending pre-order is obtained by starting with the set of ”worst” alternatives (those which are outranked by other alternatives) and going upward to the best one. The results for this example are as follows:
The descending pre-order leads to $A_2 = A_5 \succ A_3 \succ A_1 \succ A_4$.
The ascending pre-order leads to $A_2 \succ A_3 = A_5 \succ A_1 \succ A_4$.
Now, we combine the descending and ascending pre-orders to get either a complete or partial final pre-order. The level of consistency between the rankings from the two procedures determine whether the final product is a complete pre-order or a partial pre-order [4].
A commonly used method for obtaining the final pre-order is to take the intersection of the descending and ascending pre-orders. The intersection of the two pre-orders is defined such that alternative A_i outranks alternative A_j if and only if A_i outranks or is in the same class as A_j according to the two pre-orders. If alternative A_i is preferred to alternative A_j in one pre-order but A_j is preferred to A_i in the other one, then the two alternatives are incomparable in the final pre-order [4].
We get the following complete pre-order of alternatives for this example by using the above rules.

$$A_2 \succ A_5 \succ A_3 \succ A_1 \succ A_4$$

Obviously, A_2 is the best alternative at this example.

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